Statistics
Summer 2023
Lecture 15

Feb 19-8:47 AM

SE 24-27

Testing Claims:

A claim could be about

1) Population Proportion

2) Population Mean

3) Population Standard deviation

There is a claim about parameter, and we wish to test the validity of the claim.

I claim 5% of all students smoke. ← Proportion p

I claim the mean age of all students is at most 30 years. ← Mean μ

I claim the standard deviation of scores of all math exams is above 7.5. ← Standard deviation σ

Jul 10-7:31 AM

Why do we need to test a claim?

Because we want to know if claim is valid or not.

If claim is valid ⇒ we support it.

If claim is invalid ⇒ We reject it.

Possible errors:

When we reject a valid claim

When we support an invalid claim

Jul 10-7:38 AM

Testing Methods:

1) Traditional Method
2) P-value Method

we use these two methods.

3) Confidence Interval Method — use office hours to ask.

Regardless of the method, final conclusion must be the Same.

Reject the claim — when claim is invalid

OR

Support: Fail-to-Reject the claim — when claim is valid

Action \ claim	Valid	Invalid
Reject	Error	Not error
Support: Fail-to-Reject	not error	Error

Jul 10-7:43 AM

Testing Types:

1) Right-Tail Test (RTT)

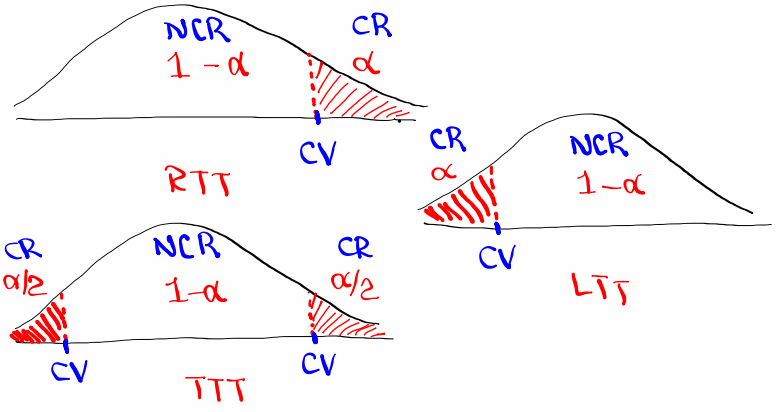
2) Left-Tail Test (LTT)

3) Two-Tail Test (TTT)

with every testing, we have a Significance level α (Alpha).

$0 < \alpha < 1$

when α not given, use .05



Jul 10-7:51 AM

Testing Process:

1) Set-up H_0 & H_1 — Alternative Hypothesis (H_a); Null Hypothesis

2) Find all Critical Values.

Drawing, Labeling, Shading, and TI Command required.

3) Find Computed Test Statistic (CTS) and P-value (P).

TI Command or formula required.

4) Use Testing Chart to determine the Validity of H_0 & H_1.

H_0 valid $\Leftrightarrow$ H_1 invalid

H_0 invalid $\Leftrightarrow$ H_1 valid

5) Draw Final Conclusion about the claim.

Claim could be H_0 or H_1.

Reject the claim when claim is invalid

OR

Fail-to-Reject FTR the claim when claim is valid

Jul 10-7:59 AM

More on H_0 & H_1:

H_0 must Contain = Sign $\Rightarrow$ $=, \geq, \leq$

H_1 Cannot Contain = Sign $\Rightarrow$ $\neq, <, >$

Key words:

H_0: is, equal, Same, at least, at most, not different, ...

H_1: is not, not equal, different, more than, less than, exceed, below, above, ...

H_1 tells us the testing Type

H_1: $\neq$ Two-Tail Test

H_1: $<$ Left-Tail Test

H_1: $>$ Right-Tail Test

Always identify the claim and Testing Types.

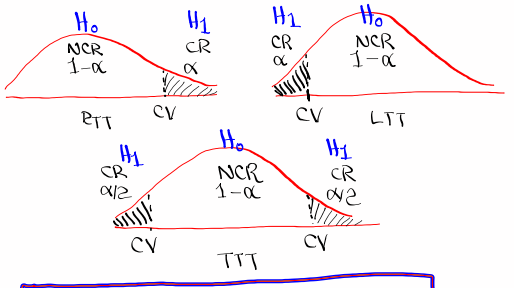
Jul 10-8:09 AM

H_0: $=$ H_1: $\neq$ TTT

H_0: $\geq$ H_1: $<$ LTT

H_0: $\leq$ H_1: $>$ RTT

Always Identify The claim



$P(H_0 \text{ valid}) = 1-\alpha = P(H_1 \text{ invalid})$

$P(H_0 \text{ invalid}) = \alpha = P(H_1 \text{ valid})$

Type I & Type II errors:

Conclusion \ Reality	H_0 valid	H_0 invalid
Support H_0	Correct Decision	Type II error
Reject H_0	Type I error	Correct Decision

Jul 10-8:30 AM

The college claims that 7.5% of all students smoke. $H_0: p = .075$ claim

$H_1: p \neq .075$ TTT

The college claims the mean age of all students is below 30 yrs. $\mu < 30$ ← H_1

$H_0: \mu \geq 30$

$H_1: \mu < 30$ claim, LTT

The college claims that standard deviation of scores of all math exams is at least 10. $\sigma \geq 10$ ← H_0

$H_0: \sigma \geq 10$ claim

$H_1: \sigma < 10$ LTT

Jul 10-8:42 AM

ESPN claims that at most 60% of all LA residents are Dodger's fan. $p \leq .6$ ← H_0

$H_0: p \leq .6$ claim

$H_1: p > .6$ RTT

LA Times has reported that the mean salary for all nurses in LA county is $6500/mo.

$H_0: \mu = 6500$ claim

$H_1: \mu \neq 6500$ TTT

I claim that stand. dev. of ages of all nurses in So. CA is below 10 yrs.

$H_0: \sigma \geq 10$

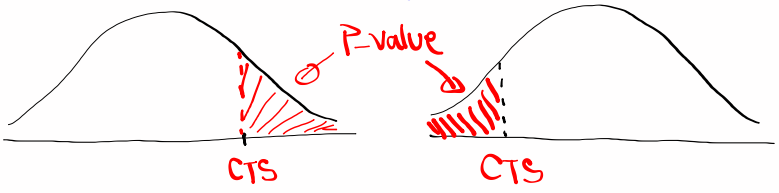
$H_1: \sigma < 10$ claim, LTT

Jul 10-8:50 AM

What is P-Value?

P-Value is the area of the tail of the graph of prob. dist. marked by CTS (Computed Test Statistics).

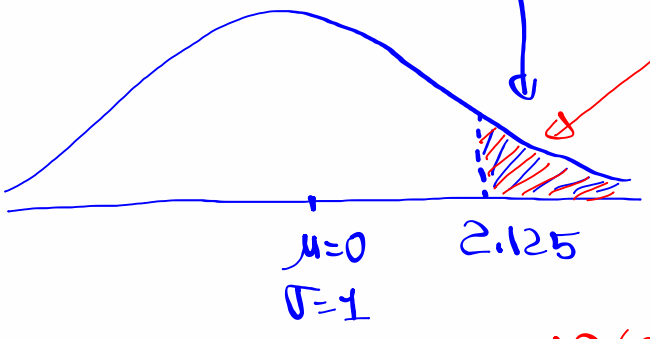
Multiply the area by 2 when Performing TTT.



only for TTT ⇒ Multiply by 2.

Jul 10-9:00 AM

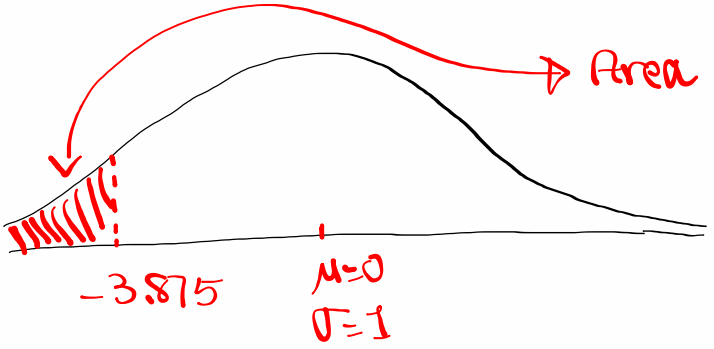
CTS $Z = 2.125$, RTT, find p-value.



P-value = normalcdf(2.125, E99, 0, 1) = .017

Jul 10-9:04 AM

Given CTS $Z = -3.875$, TTT, Find P-Value.



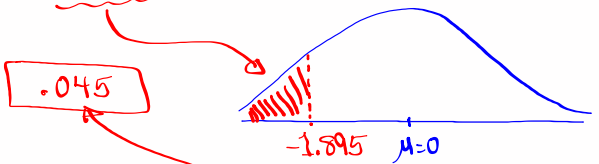
1.07×10^{-4}

P-Value = 2 * normalcdf(-E99, -3.875, 0, 1) =

Jul 10-9:06 AM

CTS $t = -1.895$, $df = 9$, LTT

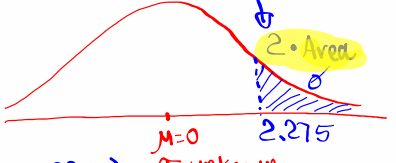
Find P-Value.



P-Value = tcdf(-E99, -1.895, 9) = .045

Given CTS $t = 2.275$, $df = 14$, TTT

Find P-Value.



P-Value = 2 * tcdf(2.275, E99, 14) = .039

SG 24 ✓

Jul 10-9:09 AM

Testing One Population Proportion: SG 25

$H_0: P = P_0$ $H_0: P \le P_0$ $H_0: P \ge P_0$

$H_1: P \ne P_0$ $H_1: P > P_0$ $H_1: P < P_0$

TTT RTT LTT

Always identify the claim

Find all critical values

Drawing, labeling, shading, TI command invNorm

Find CTS Z & P-Value P

1-PropZTest, $Z = \dfrac{\hat{P} - P}{\sqrt{\frac{PQ}{n}}}$

use normalcdf

Use Testing Chart to determine the validity of H_0 & H_1

Draw final conclusion about the claim

Reject the claim OR FTR the claim

claim is invalid claim is valid

Jul 10-9:34 AM

Given $n = 150$, $x = 18$, $\alpha = .02$

$H_0: P = .1$, claim is H_0.

Test the claim

$H_0: P = .1$ claim

$H_1: P \ne .1$ TTT

CTS $Z = .816$

P-Value $P = .414$

1-PropZTest

$P_0 = .1$ H_0

$x = 18$

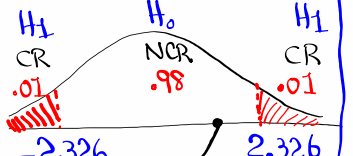
$n = 150$

Prop. $\ne P_0$ H_1

Calculate

CV Z $\alpha = .02$ TTT

use invNorm



Z = invNorm(.99, 0, 1)

CTS is in NCR

H_0 valid & H_1 invalid

P-value > α

.414 .02

Valid claim ⇒ FTR the claim

Jul 10-9:42 AM

The College claims that at most 40% of all students have a part-time job. → $p \le .4$

I Surveyed 180 students ($n = 180$) and 45% of them ($\hat{p} = .45$) had a part time job. $x = n\hat{p} = 180(.45) = 81$

Use $\alpha = .02$ to test the claim.

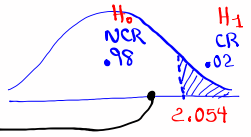
$H_0: p \le .4$ claim

$H_1: p > .4$ RTT

CTS $Z = 1.369$

P-value $P = .085$

CV Z RTT $\alpha = .02$



CV $Z = \text{invNorm}(.98, 0, 1)$

1-PropZTest

$p_0: .4$ H_0

$x = 81$

$n = 180$

Prop $> p_0$ H_1

Calculate

CTS is in NCR

H_0 valid & H_0 invalid

P-value $>$ α

.085 .02

→ Valid claim

FTR the claim

If we change $\alpha = .1$, then

P-value $\le \alpha$ ⇒ H_0 invalid & H_1 valid

.085 .1

Invalid claim ⇒ Reject the claim

Jul 10-9:53 AM

The College claims that less than 25% of all students use the tutoring Services. $p < .25$

$x = n\hat{p} = 320(.23) \approx 74$

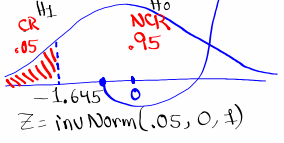
I surveyed 320 students ($n = 320$) and 23% of them ($\hat{p} = .23$) were using tutoring Services.

$H_0: p \ge .25$

Test the claim. $H_1: p < .25$ claim, LTT

CV Z LTT

NO α → USE .05



$Z = \text{invNorm}(.05, 0, 1)$

CTS $Z = -.775$

P-value $P = .219$

1-PropZTest

$p_0 = .25$ H_0

$x = 74$

$n = 320$

Prop $< p_0$ H_1

Calculate

CTS is in NCR

H_0 valid & H_1 invalid → Invalid claim

Reject the claim

P-value $>$ α

.219 .05

If we choose $\alpha = .22, .23, .24, .25, .26, \ldots$

P-value $\le \alpha$ H_0 invalid & H_1 valid

.219 $\le \alpha$

valid claim ⇒ FTR the claim

SG 25✓

Jul 10-10:07 AM

Testing One Population Mean: (SG 26)

$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$ — TTT

$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$ — LTT

$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$ — RTT

Always identify the claim

Case I: σ Known	Case II: σ unknown
CV z invNorm	CV t invT $df = n-1$
Drawing, labeling, shading, and TI command.	Drawing, labeling, shading, and TI command.
CTS z P-value P $\Rightarrow$ Z-Test inpt: Stats	CTS t P-value P $\Rightarrow$ T-Test inpt: Stats

We proceed with testing chart to learn about the validity of H_0 & H_1.

Draw Final Conclusion about the claim

Reject the claim OR FTR the claim

Jul 10-10:45 AM

Given: $n=35$, $\bar{x}=84$, $\sigma=12$

$H_0: \mu = 80$, claim is H_0, $\alpha = .1$

Test the claim

$H_0: \mu = 80$ claim

$H_1: \mu \neq 80$ TTT

with σ Known

CV z TTT $\alpha = .1$

CTS $z = -1.972$

P-value $P = .049$

CV $z = \text{invNorm}(.95, 0, 1)$

Z-Test inpt: STATS

$\mu_0: 80$

$\sigma = 12$

$\bar{x} = 84$

$n = 35$

$\mu \neq \mu_0$ H_1

Ho

CTS is in CR.

H_0 invalid & H_1 valid

P-value $\leq \alpha$

.049 .1

Invalid claim

$\Rightarrow$ Reject the claim

If we choose $\alpha = .04, .03, .02, .01$

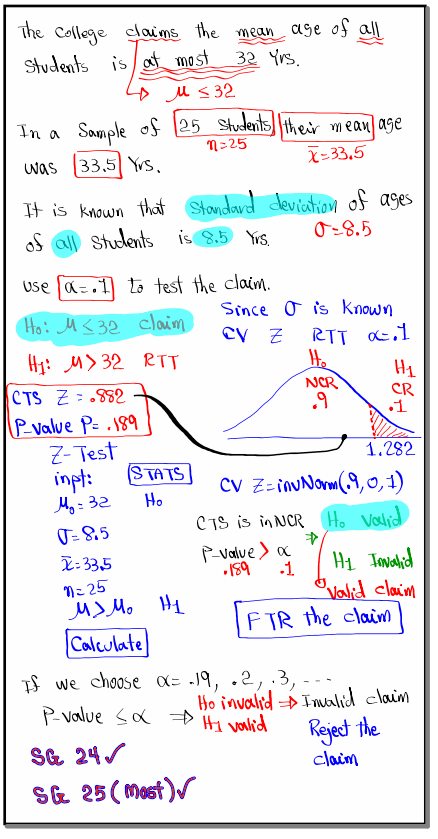
P-value $> \alpha$

.049 > .04 $\Rightarrow$ H_0 valid, H_1 invalid

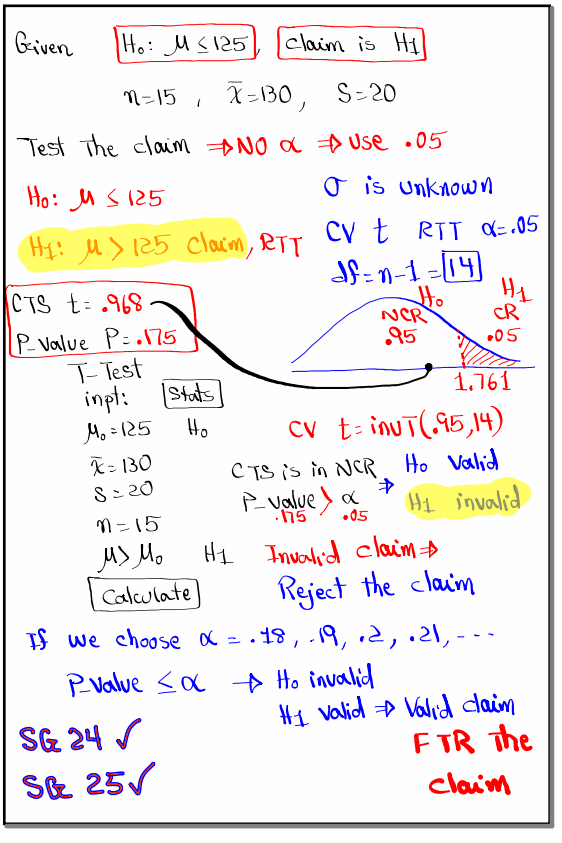
Valid claim

FTR the claim

Jul 10-10:52 AM



Jul 10-11:04 AM



Jul 10-11:25 AM